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Ninth Semiannual Status Report
December 1965

on

BASIC STUDIES IN SPACE VEHICLE ATTITUDE CONTROL

Research Grant NsG-133-61

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on

BASIC STUDIES IN SPACE VEHICLE ATTITUDE CONTROL

in the

Department of Aeronautics and Astronautics
Stanford University

under

Research Grant NsG-133-61

from the

National Aeronautics and Space Administration

This report summarizes progress during the past six months under a continuing research grant for the period beginning September 1964. The initial grant is based on Ref. 1, and its continuation on Ref. 2. The research is supervised by Prof. I. Flügge-Lotz and Prof. R. H. Cannon, Jr., Principal Investigators.

A separate financial accounting will be forwarded by the University.

SUMMARY

During this report period the study of attitude control of a spinning flexible manned space station was completed by Mr. Gevarter, and the work on random disturbances in control problems was completed by Mr. Hyver. Both Mr. Gevarter and Mr. Hyver received their Ph.D. degrees in January 1966. Five studies are continuing, three on different aspects of achieving near-optimal attitude control of planet-pointing satellites, one on a technique for trajectory optimization, and one on contributions to human pilot control. Two new studies have been started in this report period on the trajectories of orbiting vehicles.

A. STUDIES OF CONTROLLED VEHICLE BEHAVIOR
(Studies Supervised by Professor Cannon*)

1. Design of Control Systems Which Include a Human Operator (Ph.D. Research of W. G. Eppler, Jr.)

The object of a recently completed phase of this study was to determine how the performance of a human operator engaged in a closed-loop tracking task depends on which of his various outputs is used for feedback. Performance was evaluated on the basis of the closed-loop response where the desired result was to increase the bandwidth while maintaining random noise (i.e., that component of the output which is linearly uncorrelated with the input signal) at an acceptable level.

Tests were conducted in which several operators tracked a randomly varying input using (1) manual displacement, (2) muscle force, and (3) processed myoelectric (EMG) signals as the follow-up variable. The transfer function and signal-to-noise ratio were derived for each case using conventional plant-identification techniques; typical results are displayed in Fig. 1.

It can be seen that the bandwidth is increased (i.e., the phase lag at a given frequency is decreased) as the follow-up variable is changed from displacement to force and from force to processed EMG; unfortunately these bandwidth increases are accompanied by decreases in the signal-to-noise ratio. These general results were found to apply for all of the subjects tested.

The overall conclusion of this study is that the increased bandwidth of the closed-tracking loop obtained by using processed EMG as the follow-up variable probably does not warrant the increased instrumentation complexity which this approach imposes. Use of force as the follow-up variable can increase the closed-loop bandwidth as much as 30 percent (compared to displacement feedback) without seriously increasing the output noise or system complexity and should be considered for applications which do not require a steady-state controller output. It is not now planned to undertake new studies of human operator dynamics and control under this grant.

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* Professor Cannon is on sabbatical leave this year and is not being funded by NsG-133-61. He is supervising the last stages of these two research projects, however.

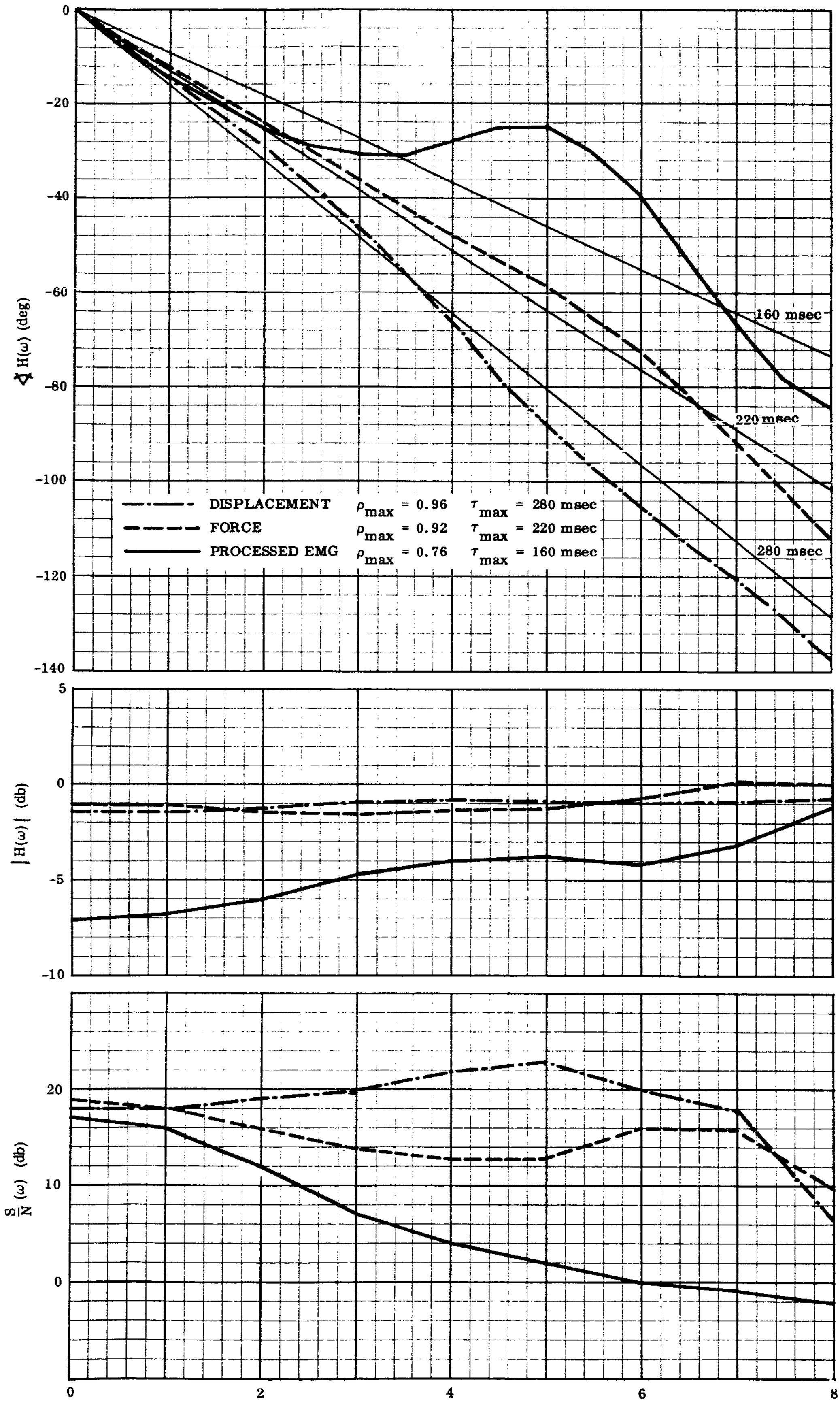

FIG. 1. TRANSFER FUNCTIONS AND SIGNAL-TO-NOISE RATIOS FOR TYPICAL SUBJECT.

2. Attitude Control of a Flexible, Spinning, Toroidal Manned Space Station (Ph.D. Research of W. B. Gevarter)

As indicated in the last status report, this study by W. B. Gevarter has considered a continuous attitude control system for controlling the direction of the spin axis of a flexible, toroidal, spinning, manned space station. Having chosen a suitable control system, assuming the vehicle rigid, the basic objective was to find locations within the space station for the control forces or moments and attitude sensors such that the space station is stable and the excitation of vibration by the control system is at a minimum. It was found that the problem is an example of a wider class of problems of the control of flexible vehicles with coupled control axes.

The validity of the simple analytical formulas discussed in Ref. 3 for determining the stability and estimating the characteristic roots of flexible vehicles with control, has been checked by the employment of a computer program to determine and plot the roots of the complete characteristic equation as the control gains are varied. It was found that excellent agreement was obtained between the computer results and the simple formulas over the range of values for which they were derived.

Applying these simple relations to the spinning space station made it possible to observe the influence of the locations of the actuators and sensors on station stability. Specifically, for a control system using simple rate and position feedback, and employing a pair of actuators about each of the orthogonal control axes, it was determined that:

1. There is no location for a single-sensor package (e.g., a star tracker and derived rate) that will yield stability of the flexural modes for control gains appropriate to the rigid-body mode.

2. The desirable solution is to use a control sensor for each control axis, and place it with one of its corresponding actuators. For this solution, the control axes are uncoupled, only the odd modes of vibration are excited, and the system is stable. Further reduction in excitation of the flexural modes can be achieved by employing lead-lag rate networks in place of pure rate.

If a single two-axis control-moment device is utilized, the system will be stable if the sensors are placed at the same location, but the flexural modes are highly excited by the control system unless filtering is employed.

The work on this problem has been completed and the results have been written up in SUDAER Report No. 250 [Ref. 4], which will be distributed shortly.

The principal contributions resulting from this investigation are as follows:

1. Development of the basic form of the equations of motion of flexible vehicles, from which the equations of forced motion of spinning pressurized toroids and the determination of their natural frequencies can be deduced.

2. Indication of a general method of approach, and derivation of simple formulas, for quickly estimating the stability, roots, and real-time response of flexible vehicles employing multi-axis linear control systems.

3. A general solution to the problem of where to place the sensors, control forces, and moments for stability of a flexible, spinning, toroidal manned space station.

4. Physical interpretation of the effect of flexibility on a control system, to provide a guide to the design or study of the control of flexible vehicles.

5. Discovery that the Coriolis forces induce precession, relative to the spinning toroid, of the natural in-plane inextensible vibrations.

A technical paper describing some of this work is now being prepared for submission to the AIAA Specialist Conference in Seattle, Washington, August 1966.

B. NONLINEAR STUDIES, OPTIMAL CONTROL
(Studies Supervised by Professor Flügge-Lotz)

1. Optimum and Suboptimal Control of the Pitch Motion of a Satellite in Elliptic Orbit; Preliminary Consideration of the Linearized Roll-Yaw Motion (Ph.D. Research of R. Busch)

The work described in the preceding status report has been continued. In particular the following points have been treated.

a. Steady-state control of the pitch motion by using a reaction wheel.

b. Control of the linearized yaw-roll system.

The equations of this system are time-varying, coupled, and difficult to analyze. However, for certain reasonable restrictions on the eccentricity of the orbit and the shape of the satellite, the equations can be simplified. An efficient suboptimal yaw-roll control system has been developed for these simplified equations. The validity of the results has been checked by comparing the phase-plane trajectories for the simplified equations and for the complete equations, in both cases by using the suboptimal control. The linear equations were simulated on the analog computer, and the nonlinear equations on a digital computer.

A report covering this yaw-roll control system and the previously mentioned pitch control system will be published next quarter.

2. The Complete Attitude Control Problem for an Earth Satellite in Elliptic Orbit (Ph.D. Research of K. Hales)

This problem and the difficulty of its solution have been described in the Eighth Semiannual Status Report (June 1965).

The acquisition problem (reduction of large attitude errors with minimum fuel control) has been considered in all details. Pontryagin's maximum principle indicates that the optimal control is a discontinuous control. With this fact in mind, a nominal path has been assumed and gradually improved by using first-order variation. A near-optimal control is achieved. Comparisons are made to known cases of optimal control (the latter were designed in backward time).

The rough draft of a report describing the solution has been finished and the final typing will soon be started. In this report, numerous examples are presented to show how well the iteration procedure works under different conditions. The examples are taken from two groups:

a. The control torque level is high enough to ignore all other disturbances such as gravity-gradient influence. An inertial reference frame can be used.

b. The control torque level is low, which means that the attitude of the satellite in a rotating reference frame has to be controlled, while the spacecraft is being disturbed by the gravity gradient.

3. The Validity of Linearization in Attitude Control
(Ph.D. Research of J.L. Almuzara)

The problem is described in detail in the preceding Status Report. At this date the comparison of the time-optimal control of

$$\left.\begin{array}{l}\ddot{x} + \sin x = u(t) \text{ and} \\ \ddot{x} + x = u(t)\end{array}\right\} \quad \text{with} \quad |u| \leq A$$

has been finished and important differences have been found. Mr. Almuzara is preparing the text of a report which will appear in the next quarter. In this report, many statements have been made concerning the more general system

$$\ddot{x} + f(x) = u(t)$$

with $f(x) = f(x + \theta)$ -- which means $f(x)$ is a general periodic function.

Mr. Frank Curtis, a new Ph.D. candidate is beginning to investigate the minimum-fuel optimization for the same system, namely $\ddot{x} + \sin x = u(t)$. This study will give us additional information about optimization of nonlinear systems. General theorems exist, but few systems are really explored far enough to give information to the designing engineer.

4. Random Disturbances in Control Problems

A report [Ref. 5] written by Mr. Gregory Hyver has been finished. Twenty-five copies will be mailed in the next days. The title of this SUDAER Report No. 246 is: "The Optimization of a Relay Controlled System Subjected to Random Disturbances."

(Studies Supervised by Professor Franklin)

5. Computation of Approximately Optimal Control (Ph.D. Research of T. E. Bullock)

This work has been concerned with investigating practical computational schemes for optimal controls. A continuation of the studies reported in the last status report has produced answers to some very important theoretical questions as well as some experimental results based on computer simulation.

One of the difficulties with the computational technique described by Merriam [Ref. 6] is that hard terminal constraints must be handled by penalty functions. During the course of our work, McReynolds and Bryson [Ref. 7] published one method for the solution of this problem; the method developed in our investigation is considerably different from that described in Ref. 7. The problem is encountered as an auxiliary problem in the study of the second variations, and concerns linear dynamics and quadratic loss. For the purpose of this note, the problem will be stated without reference to the method of second variations.

A linear plant with quadratic loss is described by the equations

$$\dot{\underline{x}} = F\underline{x} + D\underline{u} \qquad \text{dynamics}$$

$$J = \int_0^{t_f} \frac{1}{2}\left(\underline{x}'Q_1\underline{x} + \underline{u}'Q_2\underline{u}\right) dt + \frac{1}{2}\,\underline{x}'(t_f)\,Q_3\underline{x}\,(t_f) \qquad \text{cost}$$

$\underline{x}$ is $n \times 1$; $\underline{u}$ is $p \times 1$. The general linear terminal condition can be expressed as

$$A'\underline{x}(t_f) = \underline{\eta}$$

where $\underline{\eta}$ is a constant $r \times 1$ constraint vector. The problem is to find the control $\underline{u}$ to minimize cost while satisfying the terminal condition for a given initial condition,

$$\underline{x}(t_o) = \underline{x}_o$$

In this problem we assume A is a $r \times n$ matrix of rank r, $\underline{\eta}$ is $r \times 1$, $\underline{x}$ is $n \times 1$, $\underline{u}$ is $p \times 1$, and other dimensions are compatible. The usual solution leads to the two-point boundary value problem

$$\dot{\underline{x}} = F\underline{x} - DQ_2^{-1}D'\underline{\lambda}$$

Euler-Lagrange Equations

$$\dot{\underline{\lambda}} = -Q_1\underline{x} - F'\underline{\lambda}$$

with boundary conditions

$$\underline{x}(0) \text{ specified}$$

$$A'\underline{x}(t_f) = \underline{\eta}$$

and the transversality condition

$$(-Q_3\underline{x}(t_f) + \underline{\lambda}(t_f)) \in \text{Range space of } A$$

The optimal (feedback) control may be obtained from

$$\underline{u} = -Q_2^{-1}D'\underline{\lambda}(t)$$

provided $\underline{\lambda}(t)$ is known as a function of $\underline{x}(t)$.

The solution is expressed in terms of the $n \times n$ matrices $M(t)$ and $N(t)$ and a $n \times 1$ vector $\underline{b}(t)$ satisfying

$$M(t)x(t) = N(t)\lambda(t) + b(t)$$

where

$$\dot{M} = -MF - NQ$$

$$\dot{N} = NF' - MDQ_2^{-1}D'$$

$$\dot{b} = 0$$

with boundary conditions

$$M(t_f) = [(AA^+ - I)Q_2 - AA']$$

$$N(t_f) = AA^+ - I$$

$$\underline{b}(t_f) = -A\underline{\eta}$$

(The notation A^+ has been used to denote any pseudo inverse of A.)

This method requires the solution of $2n^2$ linear differential equations as opposed to the solution of $4n^2$ equations involved in the complete matrix solution to the Euler-Lagrange equations. In the special cases $r = 0$ or $r = n$, $n(n+1)/2$ nonlinear equations of the Riccati type may be solved, [Ref. 8]..

Investigation of the practical scheme of integrating the $2n^2$ equations for M, N, and b backwards until N has full rank and then switching to the normal $n(n+1)/2$ Riccati equations is underway.

(Studies Supervised by Professor Breakwell)

6. Rigorous Error Bounds on Position and Velocity in Satellite Orbit Theories (Ph.D. research of J. Vagners)

It was shown by Izsak [Ref. 9] that, to first-order in the oblateness coefficient J_2, the in-plane position and velocity components are obtainable by converting by Keplerian formulas from Brouwer's averaged Delaunay variables L', G', ℓ', g' and superimposing short-period fluctuations obtained by rewriting Brouwer's short-period generating function S_1 in terms of Hill canonical variables $(R = \dot{r}, G, r, u)$ and taking appropriate partial derivatives. These short-period fluctuations are, naturally, well-behaved (unlike those in ℓ, g) when eccentricity $e \to 0$.

Recent investigations by Vagners* have obtained in the same manner first-order "long-period fluctuations" in the Hill variables by rewriting Brouwer's long-period generating function S_1^*, relating Brouwer's L'', H G'', ℓ'', g'', h'' to L', G', H, ℓ', g', h', including general formulas for zonal harmonics. Analogous "medium-period" (i.e., daily) fluctuations in the Hill variables have been obtained in a general form for the effect of tesseral harmonics.

It is well known [Ref. 10] that the von Zeipel method of canonical transformations is a particular form of the averaging methods of Bogoliubov and Mitropolski, for which there is an associated technique for establishing bounds on the error build-up in a specified time between the exact and an approximate solution (first-order or higher-order) with the same initial conditions. Unfortunately, the bounds obtainable in this way for the Delaunay variables ℓ, g, are unsatisfactory for very small eccentricity (i.e., $e' < J_2$).

We certainly expect that error bounds on the Hill variables suffer from no such limitation. The formal von Zeipel procedure for obtaining the appropriate first-order generating functions, whether short-period, medium-period or long-period, on the other hand, is carried out necessarily in Delaunay variables.

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* Carried out under Air Force Contract AF33(615)-1411.

An analysis is in progress which parallels every transformation of Delaunay variables by an appropriate canonical transformation of the Hill variables, including removal of second-order short-period terms from the Hamiltonian. In this way it is possible to establish bounds on the first-order solution of the form $AJ_2^2+BJ_2^3t$ where A,B are independent of eccentricity e (as long as e is not close to 1).

To include the effect of inaccurate initial conditions, it suffices to examine the initial conditions of the secular Delaunay variables $L', G'', H, \ell'', g'', \ell''$, in which, as expected, an error in L' has the main effect.

7. Solar Perturbation of Mars Orbiters (Ph.D. research of R.D. Hensley)

A study is nearing completion which investigates the possible long-period fluctuations in the periapsis height of an orbiter of Mars due to the solar gravitational field. Attention is confined to orbits with low periapsis, e.g., $r_p \sim 4000 - 6000$ km (from Mars' center), and with comparatively high eccentricity, e.g., $e > 0.5$, corresponding to missions requiring, on the one hand, a close approach to the planet and, on the other hand, not excessive braking-rocket requirements. For such orbits the "secular" rotations of the orbital plane and major-axis orientation ("argument" of periapsis) due to Mars' oblateness dominate those due to the solar field. On the other hand, these oblateness secular rotations are substantially slower than Mars' motion around the sun. This latter consideration permits averaging of the perturbations, not only over orbital revolutions but also over the Martian year, to obtain equations describing "long-period" rates of change of inclination, eccentricity, longitude Ω of the node, and argument ω of periapsis, the last two elements being driven mainly by oblateness. The long-period rates of inclination and eccentricity are sinusoidal in certain linear combinations of ω and Ω, leading to fluctuations in inclination and eccentricity as sums of easily computable sinusoidal terms of known frequency. There are 11 different "critical" orbital inclinations where the frequency of a sinusoidal term vanishes with a corresponding infinitely large amplitude. A more careful analysis is developed for these "resonant" situations which reveal a finite extent to the (very slow) fluctuations in eccentricity as well as inclination. The analysis is meaningful as long as the corresponding "maximum fluctuation" in inclination does not carry it over to the vicinity of a second critical inclination. The maximum (very slow) negative deviations in periapsis height (positive deviations in eccentricity from initial e_o) correspond to particular initial conditions in ω, Ω and inclination near a critical inclination, and will be plotted vs. e_o for various $(r_p)_o$.

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